

## **Ph.D. Mathematics**

**(w.e.f. 2020-21)**

### **Program Specific Outcomes**

Students would be able to:

- PO1** Investigate emerging areas of research in mathematical sciences and study the research problems in the context of latest issues in the field.
- PO2** Review research literature, identify, formulate and solve mathematical models for the real life problems.
- PO3** Apply knowledge of research methodology and its various tools for arriving at substantiated conclusions of the research problems.
- PO4** Carry out computation and data analyses using computer software packages.
- PO5** Attain in-depth knowledge in the field of specialization and ability to conduct research which results in strengthening the discipline and its teaching.

## Scheme of Examination-Semester System for Ph.D Course Work Mathematics (w.e.f. 2020-21)

**Note:** The Scheme of Examination of Ph.D. Course Work will be the same as that for 1<sup>st</sup> Semester of M.Phil Mathematics.

### SEMESTER-I (Total Credits:14)

Paper	Title of the Course	Theory Marks	Internal-Assessment Marks	Total Marks	Hours/week	Credits
20MATPH11C1	Research Methodology	80	20	100	4	4
20MPCC1	Research and Publication Ethics	40	10	50	2	2
*	One paper either from Group A or Group B	80	20	100	4	4
*	One paper either from Group A or Group B	80	20	100	4	4
<b>TOTAL MARKS OF SEMESTER-I</b>				<b>350</b>		
*Students are required to opt both courses either from Group A or Group B						

#### Group A

20MATPH11DA1	:	Advanced Solid Mechanics
20MATPH11DA2	:	Waves and Viscoelasticity
20MATPH11DA3	:	Computational Biology
20MATPH11DA4	:	Reliability Theory
20MATPH11DA5	:	Stochastic Processes
20MATPH11DA6	:	Parametric and Non-Parametric Tests
20MATPH11DA7	:	Computational Technique-I

#### Group B

20MATPH11DB1	:	Advanced Functional Analysis
20MATPH11DB2	:	Fixed Point Theory
20MATPH11DB3	:	Fuzzy Set Theory
20MATPH11DB4	:	Wavelets-I
20MATPH11DB5	:	Sobolev Spaces-I
20MATPH11DB6	:	Algebraic Coding Theory
20MATPH11DB7	:	Algebraic Number Theory

**Note 1:** The marks of internal assessment of each course shall be split as under :

Attendance	:	05 marks
Internal Assessment Test	:	05 marks
Presentation	:	10 marks
<b>Total</b>	:	<b>20 Marks</b>

**Note 2:** The syllabus of each course will be divided into **three or four** sections of **two or three** questions each. The question paper will consist of **eight** questions divided into sections as indicated in the syllabus. The students shall be asked to attempt **five** questions selecting **atleast one** question from each section.

**Note 3:** As per UGC recommendations, the teaching program shall be supplemented by tutorials and problem solving sessions for each theory paper.

**Note 4:** Optional courses will be offered subject to availability of requisite resources/ faculty.

## 20MATPH11C1: Research Methodology

Max. Marks : 8

Credits: 4

Time: 3 Hours

### Course Outcomes

Students would be able to:

- CO1 Describe a research process and steps involved in the process.
- CO2 Understand research design and its different types.
- CO3 Distinguish between primary and secondary data and know how to collect these data using a relevant method.
- CO4 Analyse data using various charts, diagrams and statistical tools.
- CO5 Learn Microsoft Word and Microsoft Power Point Softwares.

### *Section-I (Three Questions)*

Introduction: Meaning, objectives and types of research.

Research Process: Steps involved in research process, Problems encountered by researchers in India. Research Design: Meaning and need for research design, Different research designs.

Data collection through experimental techniques and theoretical calculations, Types of data and various methods of data collection and compilation.

### *Section-II(Three Questions)*

Processing and analysis of data: Coding, editing, classification and tabulation of data, Elements of analysis, Data analysis using various kinds of charts, diagrams and statistical tools - Correlation; Fitting of curves and linear regression; Z, t, F, Chi Square tests and ANOVA.

### *Section-III(Two Questions)*

Preparation of Dissertation: Types and layout of research, Precautions in preparing the research Dissertations. Bibliography and annexure discussion of results, Drawing conclusions, giving suggestions and recommendations to the concerned persons.

Knowledge of Microsoft Word and Power point.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### Books Recommended:

1. Bill Taylor, Research Methodology: A Guide for Researchers, PHI.
2. R. P. Mishra, Research Methodology, Concept Publishing Company, New Delhi.
3. Suresh C. Shina and Anil K. Dhiman, Research Methodology, Ess, 2002
4. C. R. Kothari, Research Methodology, New Age International Publishers, 2004.

## 20MATPH11DA1: Advanced Solid Mechanics

**Max. Marks: 80**

**Credit: 4**

**Time: 3 Hours**

### Course Outcomes

Students would be able to:

- CO1** Understand the nonlinear effects in the deformation, the realistic information to be used in geophysics, earthquake engineering, civil and oil exploration studies.
- CO2** To get familiar with the basic balance laws governing the deformation.
- CO3** Interpret the pattern of deformation in various situations using Kelvin and Boussinesq problems.
- CO4** Infer the thermal effects in various physical problems for industry use.

### **Section-I (Three Questions)**

#### **Non-Linear Theory**

Deformation gradient tensor. Decomposition of a deformation stretch and rotation. Strain tensors. Strain-displacement relations. Principal stretches. Strain invariants. Length and angle changes. Deformation of volume and surface elements. Homogeneous deformation-dilation, simple extension, simple shear and plane strain. Material derivative. Velocity and acceleration fields. Principle of conservation of mass-equation of continuity. Principles of balance of linear and angular momentum. Equations of motion in spatial coordinates. Principle of conservation of energy. Piola stresses. Equations of motion in material co-ordinates.

### **Section-II (Three Questions)**

#### **General Solution of the equilibrium equations**

Papkovich-Neuber solution. Lamé's strain potential. Galerkin vector. Love's strain function. Applications to the solution of the Kelvin problem for an unbounded medium and the Boussinesq problem for a semi-infinite medium.

#### **Exact solution of some linear elastic problems**

Spherical shell subject to internal and external pressures. Gravitating elastic sphere.

### **Section-III (Two Questions)**

#### **Thermoelasticity**

Generalized Hooke's law including the effects of thermal expansion, Thermoelastic Navier's equation, Thermal stresses in a long cylindrical shell and solid cylinder, Thermal stresses in a hollow spherical shell and solid spherical shell.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended:**

1. Mal. A.K. and Singh, S.J. Deformation of Elastic Solids, Prentice Hall, 1991
2. Fung, Y.C., Foundations of Solid Mechanics.
3. S. Valliappan, Continuum Mechanics – Fundamentals, Oxford & IBH Publishing Co., 1981
4. I.S. Sokolnikoff- Mathematical Theory & Elasticity, Tata McGraw Hill, New Delhi, 1977
5. S. Saada, A.S. Elasticity: Theory and Applications, Pergamon Press, 1973.

## 20MATPH11DA2: Waves and Viscoelasticity

**Max. Marks : 80**

**Credit: 4**

**Time : 3 Hours**

### Course Outcomes

Students would be able to:

- CO1** Describe the waves on strings, their fundamental properties and mathematical formulation for different string models and Lamb's problem.
- CO2** Study liquid waves for various physical models such as deep water, canal, tank, surface of a uniform stream, interface between two liquids etc.
- CO3** Have knowledge of various viscoelastic models and their constitutive equations, and the correspondence principal of viscoelasticity and its applications.

### **Section-I (Two Questions)**

#### **Waves on Strings**

Free vibrations of an infinite string. Reflection at a change of density. Reflection at a concentrated load. Strings of finite length-normal modes. String plucked at its mid-point. String with load at its mid point. (Coulson: Waves, Secs. 13-23).

**Lamb's Problem** : A periodic line or normal point force acting on the surface of a semi-infinite elastic solid (formal solution only).

### **Section-II (Three Questions)**

#### **Liquid Waves**

Types of liquid waves, Gravity waves, Particle path, Waves in deep water, Wave energy, Rate of transmission of energy for harmonic wave, Group velocity, Effect of surface tension, Stationary waves, Waves in a canal, rectangular tank, cylindrical tank. Complex potential for a simple harmonic progressive wave, Waves on the surface of a uniform stream, Waves at the interface between fluids and effect of surface tension, Circular waves.

### **Section-III (Three Questions)**

#### **Viscoelasticity**

Spring and dashpot. Maxwell and Kelvin models. Three parameter solid. Constitutive equations for generalized Maxwell and Kelvin models. Creep compliance and relaxation modulus. Hereditary integrals. Vibrations-complex compliance, dissipation, application to specific materials, the simple spring-mass system, forced vibrations. Stress-strain relations for viscoelastic body. Correspondence principle and its application to the deformation of a viscoelastic thick-walled tube in plane strain. (Relevant Sections of Flugge's book "Viscoelasticity").

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended:**

1. Atkin, R.J. and Fox, N. An Introduction to the Theory of Elasticity.
2. Bath, M., Mathematical Aspects of Seismology, Elsevier.
3. Ben-Menahem, A. and Singh, S.J. Seismic Waves and Sources, Springer.
4. Bullen, K.E. and Bolt, A. An Introduction to the Theory of Seismology, Cambridge University Press.
5. Coulson, C.A., Waves, Longman.
6. Flugge, W., Viscoelasticity.
7. Fung, Y.C., Foundations of Solid Mechanics.
8. Besant, W. H. and Ramsey, A. S., A Treatise on Hydromechanics.

## 20MATPH11DA3: Computational Biology

**Max. Marks : 80**

**Credits: 4**

**Time : 3 Hours**

### Course Outcomes

Students would be able to:

- CO1** Understand the contents and the properties of the molecular biology, perform text and sequence based analysis and discuss the results in the light of molecular biological knowledge.
- CO2** Explain the major steps in pairwise and multiple sequences alignment and develop understanding of algorithms for them.
- CO3** Describe the principle for sequence assembly and execute pairwise sequence alignment by dynamic programming.

### **Section-I (3 Questions)**

Basic concepts of molecular biology. DNA and Proteins. The Central Dogma. Gene and Genome Sequences.

Restriction Maps - Graphs, Interval graphs. Measuring Fragment sizes.

### **Section-II (2 Questions)**

Algorithms for double digest problem (DDP) - Algorithms and complexity, approaches to DDP. Integer Programming. Partition problems. Travelling Salesman Problem (TSP) Simulated annealing.

### **Section-III (3 Questions)**

Sequence Assembly – Sequencing strategies. Assembly in practices, fragment overlap statistics, fragment alignment, sequence accuracy.

Sequence comparisons Methods - Local and global alignment. Dynamic programming method. Multiple sequence alignment.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended:**

1. M.S. Waterman, Introduction to Computational Biology, Chapman & Hall, 1995.
2. A. Baxevanis and B. Quelette, Bioinformatics, A Practical Guide to the analysis of Genes and Proteins, Wiley Interscience (1998).

## 20MATPH11DA4: Reliability Theory

**Max. Marks : 80**

**Credit: 4**

**Time : 3 Hours**

### Course Outcomes

Students would be able to:

- CO1** Understand different system structures and system redundancy used in the system reliability modeling.
- CO2** Demonstrate time dependent hazard models and system reliability models.
- CO3** Compute failure rate, mean time to failure, mean time between failures and system reliability.
- CO4** Carry out reliability and availability analysis of various systems using regenerative point and supplementary variable techniques.
- CO5** Describe economics of reliability engineering with different cost models.

### **Section-I(Three Questions)**

Reliability and Quality. Failure Data Analysis: Failure data, Failure density, Failure rate.

Some Important distributions: Exponential, Rayleigh, Weibul, Gamma and Lognormal distributions.

Laplace and Stieltjes transforms and convolutions.

Component Reliability and Hazard Models: Component reliability from test data, Mean time to failure (MTTF), Mean time between failures (MTBF), Time dependent hazard models. Bath-Tub Curve.

### **Section-II(Two Questions)**

System Reliability Models: Systems with components in series, Systems with parallel components, k-out-of-m systems, Non-series parallel systems, Systems with mixed mode failures. Standby redundancy: Simple standby system, k-out-of-n standby system.

### **Section-III(Three Questions)**

Maintainability and Availability: Maintainability function, Availability function, Reliability and availability analysis of a two-unit parallel system with repair using Markov model, Reliability and availability analysis of single-unit and two-unit cold standby systems with constant failure and repair rates using regenerative point and supplementary variable techniques.

Economics of Reliability Engineering: Manufacture's cost, Customer's cost, Reliability achievement and utility cost models, Depreciation cost models and availability cost model for parallel system.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended:**

1. E. Balagurusami, Reliability Engineering, Tata McGraw Hill, New Delhi, 1984.
2. L. S. Srinath, Reliability Engineering, Affiliated East West Press, New Delhi, 1991.
3. Elsayed A. Elsayed, Reliability Engineering, Addison Wesley Longman. Inc. Publication
4. A. Birolini, Reliability Engineering: Theory and Practical, Springer-Verlag.
5. Jai Singh Gurjar, Reliability Technology, I.K. International Publishing House Pvt. Ltd.
6. Charles E Ebeling, An Introduction to Reliability and Maintainability Engineering, Tata McGraw-Hill Publishing Company Limited, New Delhi, 2000.

## 20MATPH11DA5: Stochastic Processes

**Max. Marks : 80**

**Credit: 4**

**Time : 3 Hours**

### Course Outcomes

Students would be able to:

- CO1** Define Probability generating function (pgf) and obtain pgf's of Bernoulli, binomial, Poisson and geometric distributions.
- CO2** Compute means and variances of the probability distributions using pgf's and be expressed them in terms of Laplace Transforms.
- CO3** Understand the concept of Markov chains and can obtain higher transition probabilities.
- CO4** Demonstrate the ideas of birth and death process, immigration-emigration process, linear growth process, renewal process, Regenerative stochastic process, Markov renewal process and semi-Markov process.
- CO5** Apply the stochastic theory for modeling real systems/ phenomena and study their implications.

### **Section-I (Three Questions)**

Probability generating function: Probability generating function (pgf) of Bernoulli, binomial, Poisson and geometric distributions, Mean and variance of probability distributions using pgf.

Mean and variance of probability distributions in terms of Laplace transforms.

Stochastic Processes: definition, classification and examples.

Markov Chains: definition and examples, transition matrix, order of a Markov chain, Markov chain as graphs.

### **Section-II (Three Questions)**

Higher transition probabilities, classification of states and chains. Determination of higher transition probabilities.

Poisson Process: Introduction, postulates for Poisson process, properties of Poisson process, Poisson process and related distributions.

### **Section-III (Two Questions)**

Pure Birth process. Birth and Death process: Immigration-Emigration process, linear growth process-generating function, mean population size and extinction probability.

Definitions and simple examples of Renewal process in discrete and continuous time, Regenerative stochastic processes, Markov renewal and semi-Markov processes.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended:**

1. J. Medhi, Stochastic Processes, New Age International Publishers
2. N.T.J. Bailey, Elements of Stochastic Processes.

## 20MATPH11DA6: Parametric And Non-Parametric Tests

**Max. Marks : 80**

**Credit:4**

**Time : 3 Hours**

### Course Outcomes

Students would be able to:

- CO1** Explain various basic terms used in testing of significance.
- CO2** Understand the tests of significance and confidence intervals for single proportion, difference of two proportions, single mean, difference of two means and difference of two standard deviations in case of large samples.
- CO3** Learn about Chi-square, Students'-t and Snedcor F-statistics and their distributions and their applications in different fields.
- CO4** Apply one-way and two-way analysis of variance (ANOVA) and analysis of covariance (ANCOVA) to real life problems.
- CO5** Have knowledge of various non-parametric tests.

### **Section-I(Two Questions)**

Parameter and Statistic: Sampling distribution of a statistic, standard error and its utility.

Tests of significance: Null and alternative hypotheses, Two types of error, Critical region and level of significance, One-tailed and two-tailed tests, Critical values, Procedure for testing of hypothesis.

Large Sample Tests: Tests of significance for single proportion and single mean, for difference of two proportions, two means and two standard deviations, related confidence intervals for population parameters.

### **Section-II(Three Questions)**

Chi-square, t and F statistics : Definition and simple properties of their distributions. Chi-square tests for goodness of fit and for homogeneity for standard distributions. Contingency table, Coefficient of contingency, Test of independence. t-test for single mean, for difference of means and for observed sample correlation coefficient, F-test for equality of two population variances, related confidence intervals.

Analysis of Variance and Co-Variance: ANOVA and its basic principle, Problems on ANOVA for one-way and two-way classified data, ANOCOVA technique and its applications.

### **Section-III(Three Questions)**

Non-parametric tests: Advantages and drawbacks of non-parametric tests over parametric tests, One sample and two sample sign tests, Median test, McNemer test, Willcoxon Matched-pairs test, Rank sum tests: Wilcoxon-Mann-Whitney and Kruskal-Wallis tests, One sample runs test, Spearman's rank correlation and Kendall's coefficient of concordance tests.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended:**

1. Cramer, H. , Mathematical Methods of Statistics.
2. Mood, A. M., Graybill, F. A. and Boss, D. C., Introduction to Theory of Statistics, McGraw-Hill.
3. Goon, A. M., Gupta, M. K. and Das Gupta, B., Basic Statistics, World Press.
4. Gupta, S.C. and Kapoor, V. K., Fundamentals of Mathematical Statistics, S. Chand Pub., New Delhi.
5. C. R. Kothari, Research methodology, New Age International Publishers.

## 20MATPH11DA7: Computational Technique-I

**Max. Marks: 80**

**Credit:4**

**Time: 3 Hours**

**Course Outcomes**

Students would be able to:

**CO1** Know the basic concepts of MATLAB software.

**CO2** Understand the procedures, algorithms, and concepts required in solving specific problems.

**CO3** Code solutions to problems in MATLAB, in a legible, debug' able and efficient way.

**CO4** Create understanding of the LaTeX.

**CO5** Understand the fundamentals of LaTeX and Utilization.

### Section-I

An overview of programming, Programming language, Classification, Introduction to MATLAB Programming: Basics of MATLAB programming, Variables and assignments, Data types, Operators, working with complex numbers, Elementary Math built-in functions, operations, Strings, String functions, Cell array, Creating cell array, Introduction to Matrices in MATLAB: Defining Matrices, Matrix functions, Matrix operations,

### Section-II

Loops: For loops, while loops, Branching (conditional statements) - if statement, If else statement, Else if statement.

Functions: Executable files, Subroutines, Built-in functions and user-defined functions, Function handles, Function handles in m-files, Inline functions.

Linear Algebra: Solving a Linear System, Finding eigen values and eigenvectors.

### Section-III

Installation of the software LaTeX, Understanding Latex compilation, Basic Syntax, Writing equations, Matrix, Tables, Page Layout – Titles, Abstract Chapters, Sections, References, Equation references, citation. List making environments Table of contents, Generating new commands, Figure handling numbering, List of figures, List of tables, Generating index. Packages: Geometry, Hyperref, amsmath, amssymb, algorithms, algorithmic graphic, color, tile listing. Classes: article, book, report, beamer, slides. IEEEtran. Applications to: Writing question paper, Writing articles/ research papers, Presentation using beamer.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### Books Recommended:

1. Amos Gilat, MATLAB An Introduction with Applications 5ed, Publisher: Wiley.
2. C. F. Van Loan and K.-Y. D. Fan., Insight through Computing: A Matlab Introduction to Computational Science and Engineering, SIAM Publication, 2009.
3. Y.Kirani Singh, B.B. Chaudhari, MATLAB Programming, PHI Learning, 2007.
4. KristerAhlersten, An Introduction to Matlab, Bookboon.com.
5. RudraPratap, Getting Started with MATLAB, Oxford University Press.
6. Helmut Kopka, Patrick W. Daly, A Guide to LATEX and Electronic Publishing, 4ed, Addison-Wesley, 2004.
7. Stefan Kottwitz, Latex Beginner's Guide: Create High-Quality and Professional-Looking Texts, Articles, and Books for Business and Science Using Latex, PAKET, 2012.

## 20MATPH11DB1: Advanced Functional Analysis

Max. Marks : 80

Credit: 4

Time : 3 Hours

### Course Outcomes

Students would be able to:

- CO1 Solve problems by using Contraction Mapping and Arzela-Ascoli theorems.
- CO2 Understand the concept of normed and Banach algebras with identity, spectrum, Resolvent function and its analyticity.
- CO3 Be familiar with Gelfand's theorem, spectral radius, spectral mapping theorem and Gelfand representation for algebras with identity.
- CO4 Explain the concept of Bilinear Mappings, sesquilinear mappings and their properties.

### **Section – I (3 Questions)**

Contraction mapping theorem and its applications to differential equation, integral equation and system of linear equations. Equicontinuity, Arzla-Ascoli theorem and its application to differential equations. Weierstrass's Approximation Theorem, Stone-Weierstrass's Approximation Theorem. Semi-continuity and its applications to Arclength.

### **Section-II (3 Questions)**

Definition of normed and Banach algebras with identity. Haar measure. Regular points and spectrum. Compactness of spectrum. Resolvent function and its analyticity in the set of regular points. Gelfand's theorem about isomorphism between Banach algebras and complex numbers. Spectral radius and the spectral mapping theorem for polynomial Ideals and Maximal ideals in commutative Banach algebras with identity. The set  $C(M)$  of complex functions on the set  $M$  of maximal ideals in a Banach algebra. Gelfand representation for algebras with identity.

### **Section – III (2 Questions)**

Bilinear Mappings, Bounded bilinear mappings, sesquilinear mappings, Hermitian form, bounded sesquilinear mappings, bounded sesquilinear forms in Hilbert space.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended:**

1. Bachman, G. and LawrerieNarici, Functional Analysis, Academic Press.
2. Goffman, C. and G. Pedrick, First Course in Functional Analysis .
3. Berberian, S.K., Introduction to Hilbert Spaces, (Chelsea Publishing Co. N.Y.).
4. Babu Ram, Metric Spaces, Vinayaka Publications, New Delhi.

## 20MATPH11DB2: Fixed Point Theory

**Max. Marks : 80**

**Credit: 4**

**Time : 3 Hours**

### Course Outcomes

Students would be able to:

- CO1** Find out fixed points of locally Contractive,  $\varepsilon$ -Contractive and Contractive mappings.
- CO2** Describe Caristi Fixed Point Theorem, Convex Contraction of order n, Fixed Points and Set-Valued Mappings.
- CO3** Understand the concept of non expansive mappings, their general properties and approximation of Fixed Points.
- CO4** Use the concept of Fixed point Theorems and Contraction mappings in PM spaces.
- CO5** Apply Brouwer's Fixed point Theorem and Schauder's Theorem to solve various problems.

### **Section-I ( 3 Questions)**

Banach Contractions Principle and some consequences of Contraction Principle, A converse of contraction Principle. Retraction mappings, Computation of fixed points of locally Contractive,  $\varepsilon$ -Contractive and Contractive mappings as defined by Boyd and Wong, Caristi Fixed Point Theorem, Fixed points of local power Contraction. Local radial Contraction and Hardy Roger's type mappings in a Complete metric space, Convex Contraction of order n. Fixed Points and Set-Valued Mappings. Hyperconvex Spaces.

### **Section-II ( 3 Questions)**

Non expansive mappings, Some general properties of nonexpansive mappings. Approximation of Fixed Points of non expansive and generalized non-expansive mappings, Normal Structure, Some general properties of non expansive mappings in Hilbert and Banach spaces, Fixed points of Pseudo Contractive, Quasi nonexpansive and asymptotically nonexpansive mappings. Fixed point Theorems for mappings on PM spaces, Contraction mappings in PM spaces,  $(\varepsilon, \lambda)$  Chainable mappings Probabilistic Measure of Non-Compactness, sequence of mappings and fixed points.

### **Section-III ( 2 Questions)**

Fixed point Property, Brouwer's Fixed point Theorems and applications, Schauder's Fixed point Theorem and Consequences of Schauder's Theorem. Schauder-Tychonoff and Krasnoselkii's fixed point theorems.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### Books Recommended:

1. Istratescu, V.I., Fixed Point Theory
2. Joshi, M.C. and Bose, R.K., Some Topics in Non-linear Functional Analysis

## 20MATPH11DB3: Fuzzy Set Theory

Max. Marks : 80

Credit: 4

Time : 3 Hours

### Course Outcomes

Students would be able to:

- CO1 Understand the basic concept of Fuzzy Set Theory.
- CO2 Describe Operations and Composition of Fuzzy Relations and Fuzzy matrix.
- CO3 Familiar with Projection and Cylindrical Extension, Extension by Relation and Extension Principle.
- CO4 Get knowledge of  $\alpha$ - cut of Fuzzy Graph and Fuzzy Equivalence Relation
- CO5 Learn about Operations on General Fuzzy Numbers and properties of Fuzzy Function.

### **Section-I(3 Questions)**

Definition of Fuzzy Set, Expanding Concepts of Fuzzy Set, Standard Operations of Fuzzy Set, Fuzzy Complement, Fuzzy Union, Fuzzy Intersection, Other Operations in Fuzzy Set, T- norms and T- conorms. (Chapter 1of [1] )

Product Set, Definition of Relation, Characteristics of Relation, Representation Methods of Relations, Operations on Relations, Path and Connectivity in Graph, Fundamental Properties, Equivalence Relation, Compatibility Relation, Pre-order Relation, Order Relation, Definition and Examples of Fuzzy Relation, Fuzzy Matrix, Operations on Fuzzy Relation, Composition of Fuzzy Relation,  $\alpha$  - cut of Fuzzy Relation.

### **Section-II (3 Questions)**

Projection and Cylindrical Extension, Extension by Relation, Extension Principle, Extension by Fuzzy Relation, Fuzzy distance between Fuzzy Sets. (Chapter 2,3 of [1])  
Graph and Fuzzy Graph, Fuzzy Graph and Fuzzy Relation,  $\alpha$  - cut of Fuzzy Graph, Fuzzy Network, Reflexive Relation, Symmetric Relation, Transitive Relation, Transitive Closure, Fuzzy Equivalence Relation, Fuzzy Compatibility Relation, Fuzzy Pre-order Relation, Fuzzy Order Relation, Fuzzy Ordinal Relation, Dissimilitude Relation, Fuzzy Morphism, Examples of Fuzzy Morphism. (Chapter 4 of [1] )

### **Section-III (2 Questions)**

Interval, Fuzzy Number, Operation of Interval, Operation of  $\alpha$  - cut Interval, Examples of Fuzzy Number Operation,, Definition of Triangular Fuzzy Number, Operation of Triangular Fuzzy Number, Operation of General Fuzzy Numbers, Approximation of Triangular Fuzzy Number, Operations of Trapezoidal Fuzzy Number, Bell Shape Fuzzy Number.

Function with Fuzzy Constraint, Propagation of Fuzziness by Crisp Function, Fuzzifying Function of Crisp Variable, Maximizing and Minimizing Set, Maximum Value of Crisp Function, Integration and Differentiation of Fuzzy Function. (Chapter 5,6 of [1] )

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended:**

1. Kwang H. Lee, First Course on Fuzzy Theory and Applications, Springer International Edition, 2005.
2. H.J. Zimmerman, Fuzzy Set Theory and its Applications, Allied Publishers Ltd., New Delhi, 1991.
3. John Yen, Reza Langari, Fuzzy Logic -Intelligence, Control and Information, Pearson Education.

## 20MATPH11DB4: Wavelets –I

**Max. Marks : 80**

**Credit: 4**

**Time : 3 Hours**

### Course Outcomes

Students would be able to:

- CO1** Understand linear spaces, bases and frames, normed spaces, inner product spaces, Hilbert spaces.
- CO2** Have knowledge of trigonometric systems, trigonometric Fourier series, convergence of Fourier series and generalized Fourier series.
- CO3** Represent Fourier transforms, convolution, Plancherel formula, sampling theorem and Gibbs phenomenon.
- CO4** Demonstrate the ideas of Gabor transforms, Zak transforms and their properties.
- CO5** Be familiar with concepts and properties of Wavelet transforms.

### **Section - I (3 Questions)**

Definition and Examples of Linear Spaces, Bases and Frames, Normed Spaces, The  $L^p$  - Spaces, Definition and Examples of Inner Product Spaces, Hilbert Spaces, Orthogonal and Orthonormal Systems.

Trigonometric Systems, Trigonometric Fourier Series, Convergence of Fourier Series, Generalized Fourier Series.

### **Section - II (3 Questions)**

Fourier Transforms in  $L^1(\mathbb{R})$  and  $L^2(\mathbb{R})$ , Basic Properties of Fourier Transforms, Convolution, Plancherel Formula, Poission Summation Formula, Sampling Theorem and Gibbs Phenomenon.

Definition and Examples of Gabor Transforms, Basic Properties of Gabor Transforms.

Definition and Examples of Zak Transforms, Basic Properties of Zak Transforms, Balian- Low Theorem.

### **Section - III (2 Questions)**

Wavelet Transform, Continuous Wavelet Transforms, Basic Properties of Wavelet Transforms, Discrete Wavelet Transforms, Partial Discrete Wavelet Transforms, Maximal Overlap Discrete Wavelet Transforms.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended:**

1. K. Ahmad and F. A. Shah, Introduction to Wavelet Analysis with Applications, Anamaya Publishers, 2008.
2. Eugenio Hernandez and Guido Weiss, A first Course on Wavelets, CRC Press, New York, 1996.
3. C.K. Chui, An Introduction to Wavelets, Academic Press, 1992.
4. I. Daubechies, Ten Lectures on Wavelets, CBS-NSF Regional Conferences in Applied Mathematics, 61, SIAM, 1992.
5. Y. Meyer, Wavelets, Algorithms and Applications (translated by R.D. Rayan, SIAM, 1993).

## 20MATPH11DB5: Sobolev Spaces –I

**Max. Marks : 80**

**Credit: 4**

**Time : 3 Hours**

### Course Outcomes

Students would be able to:

- CO1** Explain the concept of test function spaces and distributions, convergence distributional derivatives.
- CO2** Define  $L^1$ -Fourier transform, Fourier transform of a Gaussian,  $L^2$ - Fourier transform, Inversion formula,  $L^p$ -Fourier transform and Convolutions.
- CO3** Learn about the spaces  $W^{1,p_\infty}(\Omega)$ ,  $W^{1,p}(\Omega)$  and their characteristic properties, density results.
- CO4** Interpret the space  $H^1(\Omega)$  and its properties, density results and Imbedding Theorems.

### **Section -I (2 Questions)**

Distributions – Test function spaces and distributions, convergence distributional derivatives.

### **Section -II (3 Questions)**

Fourier Transform –  $L^1$ -Fourier transform. Fourier transform of a Gaussian,  $L^2$ -Fourier transform, Inversion formula.  $L^p$ -Fourier transform, Convolutions.  
Sobolev Spaces - The spaces  $W^{1,p_\infty}(\Omega)$  and  $W^{1,p}(\Omega)$ . Their simple characteristic properties, density results. Min and Max of  $W^{1,p}$  – functions.

### **Section -III (3 Questions)**

The space  $H^1(\Omega)$  and its properties, density results.  
Imbedding Theorems - Continuous and compact imbeddings of Sobolev spaces into Lebesgue spaces. Sobolev Imbedding Theorem, Rellich – Kondrasov Theorem.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended :**

1. R.A. Adams, Sobolev Spaces, Academic Press, Inc. 1975.
2. S. Kesavan, Topics in Functional Analysis and Applications, Wiley Eastern Limited, 1989.
3. A. Kufner, O. John and S. Fucik, Function Spaces, Noordhoff International Publishing, Leyden, 1977.
4. A. Kufner, Weighted Sobolev Spaces, John Wiley & Sons Ltd., 1985.
5. E.H. Lieb and M. Loss, Analysis, Narosa Publishing House, 1997.
6. R.S. Pathak, A Course in Distribution Theory and Applications, Narosa Publishing House, 2001.

## 20MATPH11DB6: Algebraic Coding Theory

**Max. Marks : 80**

**Credit: 4**

**Time : 3 Hours**

### Course Outcomes

Students would be able to:

- CO1** Design the new algorithms for coding.
- CO2** Calculate the parameters of given codes and their dual codes using standard matrix and polynomial operations.
- CO3** Compare the error-detecting/correcting facilities of given codes for a given binary symmetric channel.

### **Section -I (3 Questions)**

The communication channel. The Coding Problem. Types of Codes. Block Codes. Error-Detecting and Error-Correcting Codes. Linear Codes. Hamming Metric. Description of Linear Block Codes by Matrices. Dual Codes. Hamming Codes, Golay Codes, perfect and quasi-perfect codes.

### **Section -II (3 Questions)**

Modular Representation. Error-Correction Capabilities of Linear Codes. Tree Codes. . Description of Linear Tree. Bounds on Minimum Distance for Block Codes. Plotkin Bound. Hamming Sphere Packing Bound. Varshamov-Gilbert – Sacks Bound. Bounds for Burst-Error Detecting and Correcting Codes.

### **Section -III (2 Questions)**

Convolutional Codes and Convolutional Codes by Matrices. Standard Array. Bounds on minimum distance for Convolutional Codes. V.G.S. bound. Bounds for Burst-Error Detecting and Correcting Convolutional Codes. The Lee metric, packing bound for Hamming code w.r.t. Lee metric.

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended:**

1. Ryamond Hill, A First Course in Coding Theory, Oxford University Press, 1986.
2. Man Young Rhee, Error Correcting Coding Theory, McGraw Hill Inc., 1989.
3. W.W. Peterson and E.J. Weldon, Jr., Error-Correcting Codes. M.I.T. Press, Cambridge Massachusetts, 1972.
4. E.R. Berlekamp, Algebraic Coding Theory, McGraw Hill Inc., 1968.
5. F.J. Macwilliams and N.J.A. Sloane, Theory of Error Correcting Codes, North-Holand Publishing Company.
6. J.H. Van Lint, Introduction to Coding Theory, Graduate Texts in Mathematics, 86, Springer, 1998.
7. L.R. Vermani, Elements of Algebraic Coding Theory, Chapman and Hall, 1996.

## 20MATPH11DB7: Algebraic Number Theory

**Max. Marks : 80**

**Credit: 4**

**Time : 3 Hours**

### Course Outcomes

Students would be able to:

- CO1** Understand the arithmetic of algebraic number fields.
- CO2** Prove theorems about integral bases, and about unique factorization into ideals.
- CO3** Factorize an algebraic integer into irreducibles.
- CO4** Find the ideals of an algebraic number ring.
- CO5** Understand ramified and unramified extensions and their related results.

### **Section-I (2 Questions)**

Algebraic numbers, algebraic integers, countability of set of algebraic numbers, Liouville's theorem and generalizations, transcendental numbers, algebraic number fields, Liouville's Theorem of Primitive elements, ring of algebraic integers, Theorem of Primitive Elements(Chapter 3 of book at Sr. No. 1).

### **Section-II (3 Questions)**

Norm and trace of an algebraic number, non degeneracy of bilinear pairing, existence of an integral basis, Discriminant of an algebraic number field, Ideals in the ring of algebraic integers, explicit construction of integral basis, Sign of the discriminant, cyclotomic fields, calculation for quadratic and cubic cases (Chapter 4 of book at Sr. No. 1)

### **Section-III (3 Questions)**

Integral closure, Noetherian ring, characterizing Dedekind domains, fractional ideals and unique factorization, g.c.d. and L.C.M. of Ideals, Chinese remainder theorem, Dedekind's theorem, ramified and unramified extensions. Different of an algebraic number field, factorization in the ring of algebraic integers (Chapter 5 of book at Sr. No. 1).

**Note:** The question paper will contain three sections and eight questions in all. The candidates are required to attempt five questions in all selecting at least one question from each section. All questions carry equal marks.

### **Books Recommended:**

1. Esmonde and M Ram Murty, Problems in Algebraic Number Theory, GTM Vol. 190, Springer Verlag, 1999.
2. Leveque, W.J., Topics in Number Theory – Vols. I, III Addition Wesley.
3. Narasimhan and others, Algebraic Number Theory, TIFR Pamphlet No. 4
4. Pollard, H., The Theory of Algebraic Number, Carus Monograph No. 9, Mathematical Association of America.
5. Riebenboim, P., Algebraic Numbers – Wiley Inter-science.
6. Weiss, E., Algebraic Number Theory, McGraw Hill.